The Extended Equivalence Principle and the Kramers-Kronig Relations

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Abstract

A seemingly obvious extension of the weak equivalence principle, in which all matter must respond to Post-Newtonian gravitational fields, such as Lense-Thirring and radiation fields, in a *composition-independent* way, is considered in light of the Kramers-Kronig dispersion relations for the linear response of any material medium to these fields. It is argued that known observational facts lead to violations of this extended form of the equivalence principle. (PACS numbers: 04.80.Cc, 04.80.Nn, 03.65.Ud, 67.40.Bz)

The equivalence principle lies at the foundation of General Relativity (GR); similarly, the superposition principle lies at the foundation of Quantum Mechanics (QM). There exist a profound conceptual tension between these two principles, which originates from the clash between the notion of *locality* contained within the equivalence principle, and the notion of *nonlocality* contained within the superposition principle [1]. In GR, all physical systems are to be viewed as being completely *spatially separable* into independent parts, whereas in QM, there exist entangled states (i.e., sums of product states), which lead to an intrinsic *spatial nonseparability* of certain physical systems, as evidenced by the observed violations of Bell's inequality for these systems [2]. This tension between GR and QM may lead to important experimental consequences.

Here I shall examine this tension in the restricted context of *weak* gravity interacting with *slow* matter, i.e., in the context of the *linearized* Einstein's equations for Post-Newtonian gravitational fields interacting with *nonrelativistic* material media, in particular, with macroscopically coherent quantum-mechanical matter. The resulting Maxwell-like equations lead to Lense-Thirring and gravitational radiation fields in the near zone (or induction zone) of the source, and allow us to treat the propagation of gravitational radiation through a medium placed in the near zone of the source [3].

I shall consider a seemingly obvious extension of the weak equivalence principle to such Post-Newtonian situations, which shall be called the "extended equivalence principle," i.e., that the *linear response* of the medium to weak, Post-Newtonian gravitational fields must be *independent of the composition or thermodynamic state* of the medium. This extended principle will be considered in light of the Kramers-Kronig relations. It leads to a contradiction with some known observational facts. Hence the extended equivalence principle must be a false extension of the weak equivalence principle.

For weak gravity and slow matter, Maxwell-like equations have been derived from the linearized form of Einstein's field equations [4][5][6][7]. The electric-like field $\mathbf{E}_G$, which is to be identified with the local acceleration due to gravity $\mathbf{g}$, is analogous to the electric field $\mathbf{E}$, and the magnetic-like induction field $\mathbf{B}_G$, which is to be identified with the Lense-Thirring field, is analogous to the magnetic field $\mathbf{B}$ of Maxwell's theory. The physical meaning of the field $\mathbf{E}_G \equiv \mathbf{g}$ is that it is the ordinary, local three-acceleration $\mathbf{g}$ of a given particle in a system of locally freely-falling test particles, whose motion is induced by the gravitational field as seen by an observer in a local inertial frame located at the center of mass of a

measuring system, which coincides with that of the material medium placed in the near zone of the source. The physical meaning of the magnetic-like induction field $\mathbf{B}_G$ is that it is the local angular velocity of an inertial frame centered on this test particle relative to that of the observer. The Maxwell-like equations in SI units for these weak gravitational fields in the near zone of the source, upon setting the PPN ("Parametrized Post-Newton") parameters [5] to be those of general relativity, are

$$\nabla \cdot \mathbf{D}_G = -\rho_{free} \tag{1}$$

$$\nabla \times \mathbf{E_G} = -\frac{\partial \mathbf{B}_G}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{B}_G = 0 \tag{3}$$

$$\nabla \times \mathbf{H}_G = -\,\mathbf{j}_{free} + \frac{\partial \mathbf{D}_G}{\partial t} \tag{4}$$

where $\mathbf{D}_G$ is a displacement-like field, $\mathbf{H}_G$ is a magnetic-like field intensity, ρ_{free} is the free mass density and $\mathbf{j}_{free}$ is the free mass current density of *freely falling* matter, as seen by the observer [8]. In the vacuum just outside the source, the fields $\mathbf{D}_G$ and $\mathbf{H}_G$ are related to fields $\mathbf{E}_G$ and $\mathbf{B}_G$ by the *vacuum* constitutive relations [9]

$$\mathbf{D}_G = 4\varepsilon_G \mathbf{E_G} \tag{5}$$

$$\mathbf{B}_G = \mu_G \mathbf{H}_G \tag{6}$$

where $\varepsilon_G = 1/16\pi G = 2.98 \times 10^8$ kg^2/N·m^2 and $\mu_G = 16\pi G/c^2 = 3.73 \times 10^{-26}$ m/kg [4][5][6][7][10]. Here G is Newton's constant.

For obtaining solutions in the near zone of the source, we convert these Maxwell-like equations for weak gravity into a wave equation in the standard way, and conclude that the speed of GR waves in the vacuum near the source is $c_G = \frac{1}{2}(\varepsilon_G \mu_G)^{-1/2} = 1.499 \times 10^8$ m/s, which is half the vacuum speed of light [11], and that the impedance of free space for GR waves in the vacuum near the source is $Z_G = (\mu_G/\varepsilon_G)^{1/2} = 1.12 \times 10^{-17}$ m^2/s·kg. These two quantities would correspond respectively to the vacuum speed of light $c = (\varepsilon_0 \mu_0)^{-1/2} = 2.998 \times 10^8$ m/s, and to the impedance of free space $Z_0 = (\mu_0/\varepsilon_0)^{1/2} = 377$ ohms in Maxwell's theory. Since the forms of these Maxwell-like equations are identical to those of Maxwell's apart from a sign change of the sources $-\rho_{free}$ and $-\mathbf{j}_{free}$ in the Gauss-like and the Ampere-like laws, Eqs.(1) and (4), respectively, the same boundary conditions follow from them, and

therefore the same unique solutions for corresponding electromagnetic problems carry over in a unique manner to the gravitational ones in the near zone.

Let us now place an isotropic medium in the vicinity (i.e., in the near zone) of the source, and introduce the constitutive relations for this medium

$$\mathbf{D}_G = 4K_E\varepsilon_G\mathbf{E_G} \tag{7}$$

$$\mathbf{B}_G = K_M\mu_G\mathbf{H}_G \tag{8}$$

$$\mathbf{j}_G = -\sigma_G\mathbf{E_G} \tag{9}$$

where K_E is the gravitational dielectric-like constant of the medium, K_M is its gravitational magnetic-like relative permeability, and σ_G is the gravitational analog of the electrical conductivity of the medium, whose magnitude is inversely proportional to its viscosity. Due to the sign change in the sources $-\rho_{free}$ and $-\mathbf{j}_{free}$ of the Gauss-like and Ampere-like laws, it is natural to choose to define the third constitutive relation with a minus sign, so that for *dissipative* media, σ_G is always a *positive* quantity. That this third constitutive relation should be introduced here is motivated by the fact that gravitational radiation produces a quadrupolar *shear field*, so that one might expect that the *shear viscosity* of the medium through which it propagates, should enter into the dissipation of the wave. Otherwise, there could never be any dissipation of a gravitational wave as it propagates through any medium. The phenomenological parameters K_E, K_M, and σ_G must be determined by experiment, just like the corresponding electromagnetic quantities, viz., the dielectric constant κ_e, the relative magnetic permeability κ_m, and the electrical conductivity σ_e, in Maxwell's theory.

But why is it even *permissible* to introduce nontrivial constitutive relations, Eqs.(7), (8), and (9), with $K_E \neq 1$, $K_M \neq 1$, and $\sigma_G \neq 0$? One argument *against* the introduction of such relations is that the sources of all gravitational fields are entirely determined by the masses and the mass currents of the material medium in a *composition-independent* way, since the source of spacetime curvature in Einstein's field equations arises solely from the stress-energy tensor, which includes the energy density, momentum density, and stress associated with *all* forms of matter and *all* nongravitational fields [12]. Therefore, the source of spacetime curvature must be independent of the specific composition of the source. Likewise, the response of all matter to spacetime curvature must also be independent of the

specific composition of the matter. By "composition independence," we mean here not only the independence from the specific kinds of chemical, nuclear, and elementary particle content of the matter in question (including all of the specific kinds of nongravitational interactions which bind that matter together), but also the independence from the specific thermodynamic state of the matter. For example, a drop of liquid water, or a particle of frozen ice, would both undergo exactly the same geodesic, free-fall motion in the Earth's gravitational field for the same initial conditions. In this line of reasoning, there cannot be any mysterious property of the material which would make K_E, K_M, and σ_G different for different materials. The only permissible values of these constants would then be their vacuum values, viz., $K_E = 1$, $K_M = 1$, and $\sigma_G = 0$. Otherwise, there would exist a seeming violation of the equivalence principle, since there would then exist a *composition-dependent* response of different kinds of materials to gravitational fields.

However, one must carefully distinguish between the *weak* equivalence principle, where the composition independence of the *free-fall response* of matter to gravity has been extensively experimentally tested in the low-frequency, Newtonian-gravity limit, and the *extended* equivalence principle, which extends the composition independence of the weak equivalence principle to include the *linear response* of all kinds of matter to Post-Newtonian gravitational fields, such as Lense-Thirring and gravitational radiation fields. Because of the difficulty of generating and detecting these Post-Newtonian fields, this extended form of the equivalence principle has not been extensively experimentally tested.

The above Maxwell-like equations, in conjunction with the constitutive relations, Eqs.(7), (8), and (9), with $K_E \neq 1$, $K_M \neq 1$, and $\sigma_G \neq 0$, lead to the near-zone propagation in the medium of a gravitational plane wave at a frequency ω at a phase velocity $v_{phase}(\omega)$ given by$v_{phase}(\omega) = c_G/n_G(\omega)$, where the index of refraction $n_G(\omega)$ of the medium is given by$n_G(\omega) = (K_E(\omega)K_M(\omega))^{1/2}$. Such a solution is formally identical to that for an electromagnetic plane wave propagating inside a dispersive optical medium, whose index of refraction is given by $n(\omega) = (\kappa_e(\omega)\kappa_m(\omega))^{1/2}$. Due to the validity of the linearity approximation in the weak gravity limit, induction-zone fields can always be Fourier decomposed into a superposition of such plane waves, which form a complete basis set.

Since we are considering the *linear response* of the medium to *weak* gravitational radiation fields, and since the response of the medium must be *causal*, the index of refraction $n_G(\omega)$

must obey the Kramers-Kronig relations [13]

$$\text{Re } n_G(\omega) - 1 = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\text{Im } n_G(\omega')}{\omega' - \omega} d\omega' \tag{10}$$

$$\text{Im } n_G(\omega) = -\frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\text{Re } n_G(\omega') - 1}{\omega' - \omega} d\omega', \tag{11}$$

where P denotes the Cauchy Principal Value. The zero-frequency sum rule follows from the first of these relations, viz.,

$$\text{Re } n_G(\omega \to 0) = 1 + \frac{c}{\pi} \int_{0}^{+\infty} \frac{\alpha_G(\omega')}{(\omega')^2} d\omega', \tag{12}$$

where $\alpha_G(\omega)$ is the power attenuation coefficient of the gravitational plane wave at frequency ω propagating through the medium, i.e., $\exp(-\alpha_G(\omega)z)$ is the exponential factor which attenuates the power of a wave propagating along the z axis. For media in the ground state, the integrand on the right-hand side of this zero-frequency sum rule is always positive definite. A nonvanishing dissipation coefficient $\sigma_G(\omega)$ introduced in conjunction with the third constitutive relation will lead to a nonvanishing, positive value of $\alpha_G(\omega)$. Since the medium is in its ground state, the gravity wave cannot grow exponentially with propagation distance z. Hence $\alpha_G(\omega) > 0$ for all frequencies ω, and therefore Re $n_G(\omega \to 0) > 1$.

Now suppose that the extended equivalence principle were true. Then the response of any medium to gravitational radiation at all frequencies must be characterized by $K_E(\omega) = 1$ and $K_M(\omega) = 1$, independent of the composition of the medium. In particular at low frequencies, this implies that the index of refraction $n_G(\omega \to 0) = 1$ must strictly be unity. It then follows from the above zero-frequency sum rule, that the attenuation coefficient $\alpha_G(\omega) = 0$ must strictly vanish for all frequencies ω. This result would imply that *any* absorption of gravitational radiation in the near zone of the source would be impossible at *any* frequency by *any* kind of matter. Detectors of gravitational radiation would be impossible. By reciprocity, emission of gravitational radiation by any kind of matter at any frequency would likewise also be impossible. Gravitational radiation might as well not exist [14]. This, however, is contradicted by the observations of Taylor and Weisberg [15]. Thus the extended equivalence principle must be a false extension of the weak equivalence principle.

Since we are dealing with the linear response of a medium placed in the *near zone* of the source, it may be objected that the concept of *index of refraction*, which usually refers to a

medium placed in the far zone (or radiation zone) of the source, has been used in the above argument. In fact, the use of the far-zone approximation is not necessary here; the concept of index of refraction *can* be a valid one for the treatment of the interaction of a medium with radiation fields in the near zone of a planar source, for example, in the response of a thin, dissipative film to a plane wave, when the film is placed parallel to this planar source and is separated from it by a distance much less than a wavelength [1].

Moreover, it is possible to recast the above argument in terms of the Kramers-Kronig relations for the real and imaginary parts of the dielectric-like constant K_E and of the relative permability-like constant K_M, which are valid concepts for the treatment of the near-zone response of the medium. No concept of *wave propagation* is necessarily involved here when one only uses K_E and K_M separately in the treatment of the medium's *linear* and *causal* response to Post-Newtonian gravitational fields. There result from the separate Kramers-Kronig relations for K_E and K_M the two separate zero-frequency sum rules [16][17]

$$\text{Re } K_E(\omega \to 0) = 1 + \frac{2}{\pi}\int_0^{+\infty} \frac{\text{Im } K_E(\omega')}{\omega'} d\omega', \tag{13}$$

$$\text{Re } K_M(\omega \to 0) = 1 + \frac{2}{\pi}\int_0^{+\infty} \frac{\text{Im } K_M(\omega')}{\omega'} d\omega', \tag{14}$$

where $\omega\text{Im } K_E(\omega)$ is proportional to the power absorption coefficient due to the imaginary part of $K_E(\omega)$, and $\omega\text{Im } K_M(\omega)$ is proportional to the power absorption coefficient due to the imaginary part of $K_M(\omega)$. Again, for a medium in the ground state, the integrands on the right-hand sides are positive definite. The above argument repeated for these sum rules leads to the absurd conclusion that there is *no* possibility for *any* absorption of energy by *any* medium in its response to gravitational fields at *any* frequency, independent of the composition of the medium.

Hence, not only is it *permissible*, but it is also *necessary*, to introduce constitutive equations with nontrivial, composition-dependent values of K_E, K_M, and σ_G. In the case of electromagnetism, we know that nontrivial values of the relative permeability $\kappa_m \neq 1$ are only possible due to purely quantum-mechanical effects, viz., paramagnetism and ferromagnetism, due to the alignment of electron spin, and diamagnetism, such as the Meissner effect in superconductors, due to London's rigidity of the macroscopic condensate wavefunction of Cooper pairs of electrons. Likewise, one expects here that nontrivial values of the gravitational relative permeability $K_M \neq 1$ can also only be possible due to purely quantum-mechanical effects.

There may already be experimental evidence of the existence of a strong Meissner-like effect in the case of superfluid helium, which can be interpreted as evidence of a gross, quantum violation of the extended equivalence principle. The Hess-Fairbank effect [18] is an analog of the Meissner effect observed in a slowly rotating bucket of liquid helium, as it is slowly cooled through the normal-superfluid (or lambda) transition temperature, in which the angular momentum of the rotating normal fluid is *expelled* from the superfluid. This may be viewed as evidence for a true Meissner-like effect, in which the Lense-Thirring field arising from distant matter in the Universe is expelled. From an application of a limited form of Mach's principle in GR using the Lense-Thirring effect from distant, rotating cylindrical shells of stars [12], it follows that the parabolic shape of the surface of any steadily rotating bucket of *any* classical fluid, after it has reached mechanical equilibrium, is described by $y = \frac{1}{2}\Omega^2 x^2/g$, where x is the distance from the bucket's axis of rotation of a fluid element on the surface, y is the height of this fluid element with respect to the height of the liquid surface at $x = 0$, Ω is the bucket's rate of rotation with respect to the fixed stars, and g is the acceleration due to Earth's gravity. This result, which is a consequence of the equivalence principle, is independent of the composition of the liquid. For example, steadily rotating buckets filled with mercury, or water, or normal liquid helium above its superfluid transition temperature, will all come to the same parabolic shape of surface for the same Ω, independent of atomic composition.

One implication of the Hess-Fairbank effect is that the parabolic surface of a bucket of rotating normal liquid helium should become flat below the thermodynamic transition from the normal to the superfluid state. It is possible to measure the curvature of the surface of the rotating fluid by reflecting a laser beam from it. The flattening of the surface below the lambda transition temperature of liquid helium in a slowly rotating bucket, as it is cooled slowly enough so that it always stays in thermal equilibrium throughout the normal-to-superfluid thermodynamic phase transition, has been observed by the use of laser optical systems and laser interferometry [19]. Since the flattened-surface response of *superfluid* helium to the Lense-Thirring field of the distant matter of the Universe is clearly different from the parabolic-surface response of any *classical* fluid, this effect should be viewed as evidence for a macroscopic quantum violation of the extended equivalence principle.

For all of the above reasons, one concludes that the extended equivalence principle is an invalid generalization of the weak equivalence principle.

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generation of gravitational waves (GW) by any slowly moving mass in the induction zone. However, since it assumes that any velocities within the mass moves slowly compared to c, and since GWs propogate at c, it cannot consistently incorporate GWs into its formalism in the radiation zone, but only in the induction zone, where the metric is overwhelmingly determined by the generating mass. However, the correct value of c for radiation-zone GW propagation comes out of a similar set of vacuum Maxwell-like equations, which have been derived without making the PPN approximation (I thank A. D. Speliotopoulos for pointing this out to me).

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